647510 **BSCbooCC5030** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
Feb/Mar. -2021 (OLD COURSE)
BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1(CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(A) Answer any one. (07)

1. State and prove distributive inequality and modular inequality for lattices.
2. Prove that direct product of two lattices is a lattice. Draw Hasse diagram of $S_4 \times S_9$, where S_n (the set of all divisors of natural number n) is a lattice with divides relation.

Q.1(B) Answer any one. (04)

1. Prove that every chain is a distributive lattice.
2. State and prove De Morgan's laws for a lattice.

Q.1(C) Answer any one. (03)

1. Let $(L, \leq)$ be a lattice. Prove that $a * (a \oplus b) = a$, $a, b \in L$.
2. Draw Hasse diagram of (S_{60}, D).

Q.2(A) Answer any one. (07)

1. Let $(B, *, \oplus, ' \ 0, 1)$ be a Boolean algebra. In usual notations prove that
   (A) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$, for all $x_1, x_2 \in B$.
   (B) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$, for all $x_1, x_2 \in B$.
2. State and prove Stone's representation theorem.

Q.2(B) Answer any one. (04)

1. Let $(B, *, \oplus, ', 0, 1)$ be a Boolean algebra. Prove that $a \in B$ is an atom if and only if for any $x \in B$, either $a * x = 0$ or $a * x = a$.
2. Using Karnaugh map. Reduce the following Boolean expressions.
   (A) $xy + xy'$ (B) $xy + x'y + x'y'$

Q.2(C) Answer any one. (03)

1. Find sum of product canonical form of Boolean expression $\alpha(x_1, x_2, x_3) = x_1 \oplus x'_2$.
2. Find cube array representation of $f(x, y, z) = xy + xz'$.

Q.3(A) Answer any one. (07)

1. State and prove sufficient condition for a function to be analytic.
2. If a function $f(z) = u + iv$ is analytic then prove that u and v are harmonic functions. Verify the converse of this result for $f(z) = e^{\bar{z}}$.

Q.3(B) Answer any one. (04)

1. Show that the function $f(z) = \frac{\bar{z}^2}{z}$; $z \neq 0$, $f(0) = 0$ is not differentiable at origin although Cauchy-Reimann equations are satisfied at origin.
2. Find harmonic conjugate of $v(x, y) = 3x^2y - y^3$. Also find the analytic function $f(z) = u + iv$.

Q.3(C) Answer any one. (03)

1. Prove that $f(z) = \bar{z}$ is nowhere differentiable.
2. Show that $f(z) = \cos z$ is an entire function.

Q.4(A) Answer any one. (07)

1. Derive Laplace equation in polar form.
2. State and prove Cauchy's integral formula.

Q.4(B) Answer any one. (04)

1. State and prove Cauchy-Reimann equations in polar form.
2. Evaluate $\int_C \frac{z}{(z-1)(z-2)}\, dz$, where C is the circle.

   (A) $|z| = \frac{1}{2}$. (B) $|z| = 1$.

Q.4(C) Answer any one. (03)

1. Evaluate $\int_0^{2+i} (\bar{z})^2\, dz$ along the real axis to 2 and then vertically to $2 + i$.
2. Evaluate $\int_C \frac{\cos z}{z(z^2+8)}\, dz$, where C is the positively oriented boundary of the square whose sides lie along $x = \pm 2$ and $y = \pm 2$.

Q.5(A) Answer any one. (07)

1. State and prove Morera's theorem.
2. State and prove fundamental theorem of algebra.

Q.5(B) Answer any one. (04)

1. Evaluate $\int_C \frac{1}{(z^2+4)^2}\, dz$, where C is the circle $|z - i| = 2$.
2. State and prove Liouville's theorem.

Q.5 (C) Answer any one. (03)

1. Let C be the are of the circle $|z| = 2$ lying in the first quadrant. Show that $\int_C \frac{z+4}{(z^2-1)}\, dz \le \frac{6\pi}{7}$.
2. State and prove Cauchy's inequality.

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